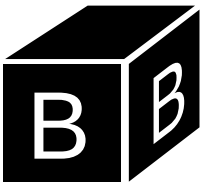
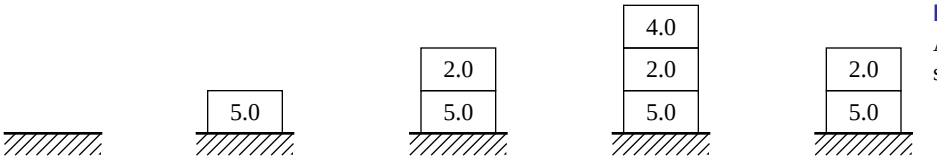
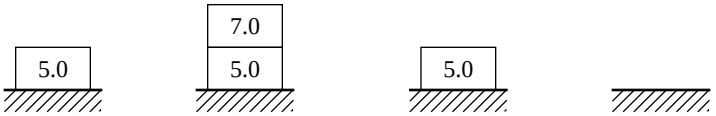


Chapter 7

Abstract Stacks and Lists



(a) BEGIN **(b)** Push(5.0) **(c)** Push(2.0) **(d)** Push(4.0) **(e)** Pop(d.x)



(f) Pop(d.x) **(g)** Push(7.0) **(h)** Pop(d.x) **(i)** Pop(d.x)

Figure 7.1
A sequence of operations on a stack.

There are three kinds of arithmetic notation:

- Infix 3 + 5
- Prefix + 3 5
- Postfix 3 5 +

```
DEFINITION PboxStackADS;

    CONST
        capacity = 8;

    PROCEDURE Clear;
    PROCEDURE NumItems (): INTEGER;
    PROCEDURE Pop (OUT val: REAL);
    PROCEDURE Push (val: REAL);

END PboxStackADS.
```

Figure 7.2
The interface of the stack abstract data structure.

CONST **capacity**
The maximum number of real values in the stack.

PROCEDURE **Clear**
Post
The stack is cleared to the empty stack.

PROCEDURE **NumItems** (): INTEGER
Post
Returns the number of elements in the stack.

PROCEDURE **Pop** (OUT val: REAL)
Pre
0 < NumItems() 20
Post
An item is removed from the top of the stack and val gets its value.

PROCEDURE **Push** (val: REAL)
Pre
NumItems() < capacity 20
Post
val is pushed onto the top of the stack.

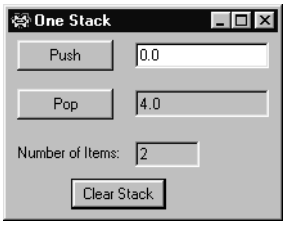
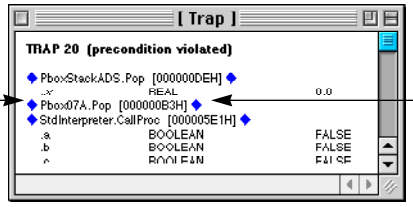


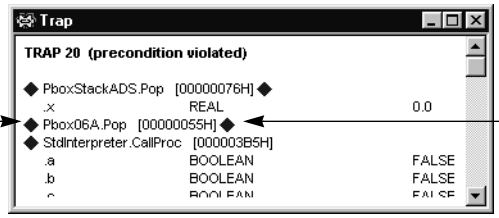
Figure 7.3
The dialog box that goes with the program in Figure 7.4.

```
MODULE Pbox07A;
   IMPORT Dialog, PboxStackADS;

   VAR
      d*: RECORD
         valuePushed*, valuePopped-: REAL;
         numItems-: INTEGER;
      END;

   PROCEDURE Push*;
   BEGIN
      PboxStackADS.Push(d.valuePushed);
      d.numItems := PboxStackADS.NumItems();
      Dialog.Update(d)
   END Push;

   PROCEDURE Pop*;
   BEGIN
      PboxStackADS.Pop(d.valuePopped);
      d.numItems := PboxStackADS.NumItems();
      Dialog.Update(d)
   END Pop;

   PROCEDURE ClearStack*;
   BEGIN
      PboxStackADS.Clear;
      d.valuePushed := 0.0; d.valuePopped := 0.0;
      d.numItems := 0;
      Dialog.Update(d)
   END ClearStack;

BEGIN
   ClearStack
END Pbox07A.
```

Figure 7.4
A program that uses the stack abstract data structure.

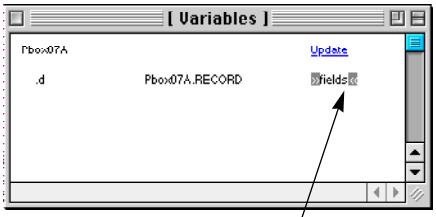


(a) MacOS



(b) MSWindows

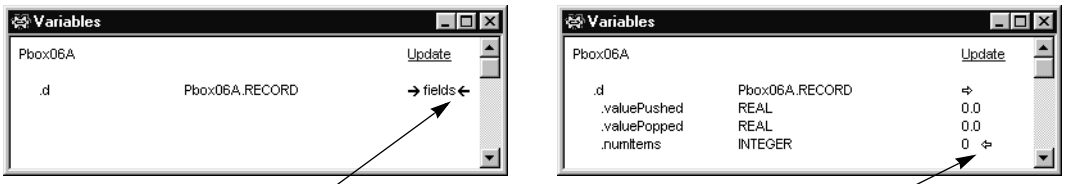
Figure 7.5
A trap window for the stack ADS program.



Fold mark for collapsed fold

Fold mark for expanded fold

(a) MacOS



Fold mark for collapsed fold

Fold mark for expanded fold

(b) MSWindows

Figure 7.6
Global variables generated from the trap window of Figure 7.5.

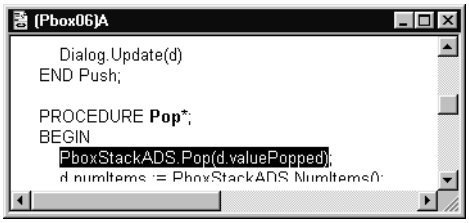


Figure 7.7
Source code window opened by the link to the source code in Figure 7.5.

```
DEFINITION PboxStackADT;

    CONST
        capacity = 8;

    TYPE
        Stack = RECORD END;

    PROCEDURE Clear (VAR s: Stack);
    PROCEDURE NumItems (IN s: Stack): INTEGER;
    PROCEDURE Pop (VAR s: Stack; OUT val: REAL);
    PROCEDURE Push (VAR s: Stack; val: REAL);

END PboxStackADT.
```

Figure 7.8
The interface of the stack abstract data type.

TYPE **Stack**
The stack abstract data type supplied by PboxStackADT.

PROCEDURE **Clear** (VAR s: Stack)
Post
Stack s is cleared to the empty stack.

PROCEDURE **Pop** (VAR s: Stack; OUT val: REAL)
Pre
0 < NumItems(s) 20
Post
An item is removed from the top of stack s and val gets its value.

- Default Call by value
- IN Call by constant reference
- OUT Call by result
- VAR Call by reference

The four calling modes of Component Pascal

- In call by value (default) and call by constant reference (IN), the procedure does *not* change the value of the actual parameter.
- In call by result (OUT) and call by reference (VAR), the procedure *does* change the value of the actual parameter.

Changing versus not changing the value of the actual parameter.

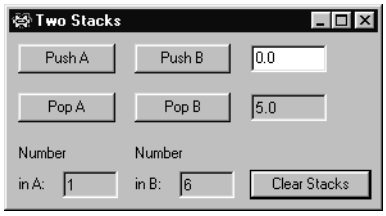


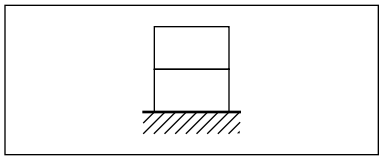
Figure 7.9
The dialog box for manipulating two stacks.

```
MODULE Pbox07B;
   IMPORT Dialog, PboxStackADT;

   VAR
      d*: RECORD
         valuePushed*, valuePopped-: REAL;
         numItemsA-, numItemsB-: INTEGER;
      END;
      stackA, stackB: PboxStackADT.Stack;

   PROCEDURE PushA*;
   BEGIN
      PboxStackADT.Push(stackA, d.valuePushed);
      d.numItemsA := PboxStackADT.NumItems(stackA);
      Dialog.Update(d)
   END PushA;

   PROCEDURE PushB*;
   BEGIN
      PboxStackADT.Push(stackB, d.valuePushed);
      d.numItemsB := PboxStackADT.NumItems(stackB);
      Dialog.Update(d)
   END PushB;
```

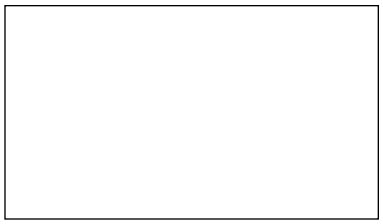
Figure 7.10
A program that uses the stack abstract data type.

```
    PROCEDURE PopA*;
    BEGIN
        PboxStackADT.Pop(stackA, d.valuePopped);
        d.numItemsA := PboxStackADT.NumItems(stackA);
        Dialog.Update(d)
    END PopA;

    PROCEDURE PopB*;
    BEGIN
        PboxStackADT.Pop(stackB, d.valuePopped);
        d.numItemsB := PboxStackADT.NumItems(stackB);
        Dialog.Update(d)
    END PopB;

    PROCEDURE ClearStacks*;
    BEGIN
        PboxStackADT.Clear(stackA);
        PboxStackADT.Clear(stackB);
        d.valuePushed := 0.0; d.valuePopped := 0.0;
        d.numItemsA := 0; d.numItemsB := 0;
        Dialog.Update(d)
    END ClearStacks;

BEGIN
    ClearStacks
END Pbox07B.
```

PboxStackADS

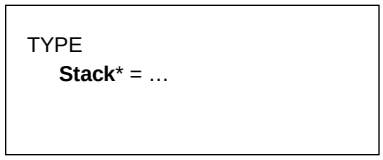


Pbox07A

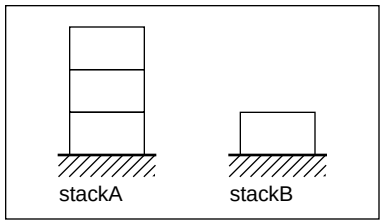


(a) Abstract data structure

PboxStackADT



Pbox07B



(b) Abstract data type

Figure 7.11
The difference between an abstract data structure and an abstract data type.

0	trout
1	tuna
2	cod
3	salmon
4	
5	
6	
7	

Figure 7.12
A list that contains a maximum of eight strings.

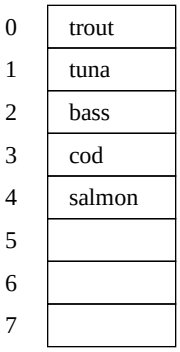
```
DEFINITION PboxListADS;

    CONST
        capacity = 8;
    TYPE
        T = ARRAY 16 OF CHAR;

    PROCEDURE Clear;
    PROCEDURE Display;
    PROCEDURE GetElementN (n: INTEGER; OUT val: T);
    PROCEDURE InsertAtN (n: INTEGER; IN val: T);
    PROCEDURE Length (): INTEGER;
    PROCEDURE RemoveN (n: INTEGER);
    PROCEDURE Search (IN srchVal: T; OUT n: INTEGER; OUT fnd: BOOLEAN);

END PboxListADS.
```

Figure 7.13
The interface of the list abstract data structure.

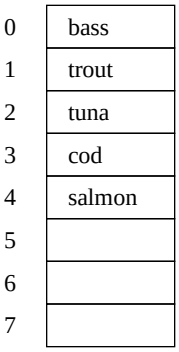
PROCEDURE **Clear**
Post
The list is initialized to the empty list.

PROCEDURE **Length** (): INTEGER
Post
Returns the number of elements in the list.

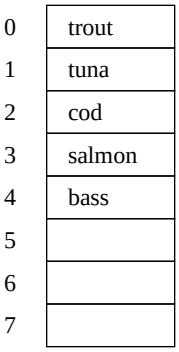
PROCEDURE **InsertAtN** (n: INTEGER; IN val: T)
Pre
0 <= n 20
Length() < capacity 21
Post
val is inserted at position n in the list, increasing Length() by 1.
If n > Length(), val is appended to the list.



(a) After inserting at position 2.



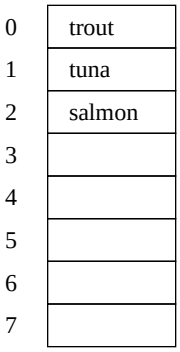
(b) After inserting at position 0.



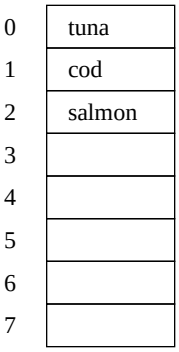
(c) After inserting at position 7.

Figure 7.14
The list of Figure 7.12 after executing the statements of Example 7.9.

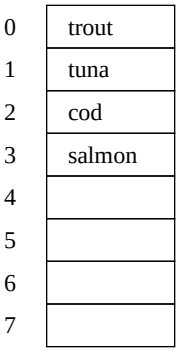
PROCEDURE **RemoveN** (n: INTEGER)
Pre
0 <= n 20
Post
If n < Length(), the element at position n in the list is removed.
Otherwise, the list is unchanged.



(a) After removing from position 2.



(b) After removing from position 0.



(c) After removing from position 7.

Figure 7.15
The list of Figure 7.12 after executing the statements of Example 7.10.

PROCEDURE **GetElementN** (n: INTEGER; OUT val: T)
Pre
0 <= n 20
n < Length() 21
Post
val gets the data value of the element at position n of the list.
Note: 0 is the position of the first element in the list.

PROCEDURE **Search** (IN srchVal: T; OUT n: INTEGER; OUT fnd: BOOLEAN)
Post
If srchVal is in the list, fnd is set to TRUE and n is set to the first position where srchVal is found.
Otherwise, fnd is set to FALSE and n is undefined.

PROCEDURE **Display**
Post
The list is output to the Log, one element per line with each element preceded by its position.

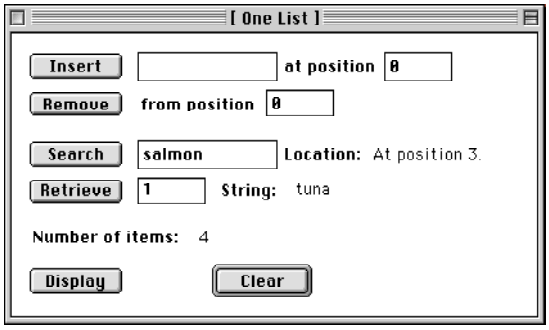


Figure 7.16
The dialog box for manipulating a list.

```
MODULE Pbox07C;
    IMPORT Dialog, PboxListADS, PboxStrings;

    TYPE
        String32 = ARRAY 32 OF CHAR;

    VAR
        d*: RECORD
            insertT*: PboxListADS.T; insertPosition*: INTEGER;
            removePosition*: INTEGER;
            searchT*: PboxListADS.T; searchPosition-: String32;
            retrievePosition*: INTEGER; retrieveT-: PboxListADS.T;
            numItems-: INTEGER;
        END;

    PROCEDURE InsertAt*;
    BEGIN
        PboxListADS.InsertAtN(d.insertPosition, d.insertT);
        d.numItems := PboxListADS.Length();
        Dialog.Update(d)
    END InsertAt;

    PROCEDURE RemoveFrom*;
    BEGIN
        PboxListADS.RemoveN(d.removePosition);
        d.numItems := PboxListADS.Length();
        Dialog.Update(d)
    END RemoveFrom;
```

Figure 7.17
A program that uses the list abstract data structure to implement the dialog box of Figure 7.16.

```
    PROCEDURE SearchFor*;
    VAR
        found: BOOLEAN;
        position: INTEGER;
    BEGIN
        PboxListADS.Search(d.searchT, position, found);
        IF found THEN
            PboxStrings.IntToString(position, 1, d.searchPosition);
            d.searchPosition := "At position " + d.searchPosition + "."
        ELSE
            d.searchPosition := "Not in list."
        END;
        Dialog.Update(d)
    END SearchFor;

    PROCEDURE RetrieveFrom*;
    BEGIN
        PboxListADS.GetElementN(d.retrievePosition, d.retrieveT);
        Dialog.Update(d)
    END RetrieveFrom;

    PROCEDURE DisplayList*;
    BEGIN
        PboxListADS.Display;
        Dialog.Update(d)
    END DisplayList;

    PROCEDURE ClearList*;
    BEGIN
        PboxListADS.Clear;
        d.insertT := ""; d.insertPosition := 0;
        d.removePosition := 0;
        d.searchT := ""; d.searchPosition := "";
        d.retrievePosition := 0; d.retrieveT := "";
        d.numItems := PboxListADS.Length();
        Dialog.Update(d)
    END ClearList;

BEGIN
    ClearList
END Pbox07C.
```

```
DEFINITION PboxListADT;

    CONST
        capacity = 8;

    TYPE
        List = RECORD END;
        T = ARRAY 16 OF CHAR;

    PROCEDURE Clear (VAR lst: List);
    PROCEDURE Display (IN lst: List);
    PROCEDURE GetElementN (IN lst: List; n: INTEGER; OUT val: T);
    PROCEDURE InsertAtN (VAR lst: List; n: INTEGER; IN val: T);
    PROCEDURE Length (IN lst: List): INTEGER;
    PROCEDURE RemoveN (VAR lst: List; n: INTEGER);
    PROCEDURE Search (VAR lst: List; IN srchVal: T; OUT n: INTEGER; OUT fnd: BOOLEAN);

END PboxListADT.
```

Figure 7.18
The interface of the list abstract data type.

PROCEDURE **InsertAtN** (n: INTEGER; IN val: T)
Pre
0 <= n 20
Length() < capacity 21
Post
val is inserted at position n in the list, increasing Length() by 1.
If n > Length(), val is appended to the list.

PROCEDURE **InsertAtN** (VAR lst: List; n: INTEGER; IN val: T);
Pre
0 <= n 20
Length(lst) < capacity 21
Post
val is inserted at position n in list lst, increasing Length(lst) by 1.
If n > Length(lst), val is appended to lst.

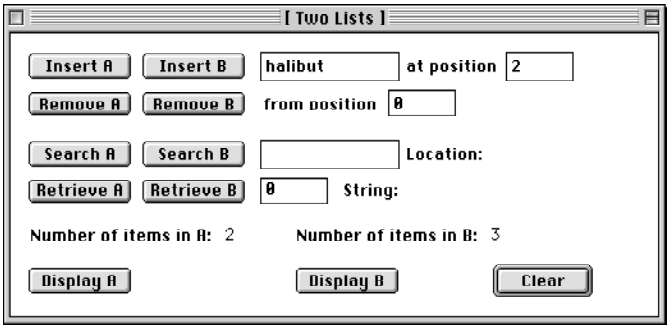


Figure 7.19
The dialog box for manipulating two lists.

```
MODULE Pbox07D;
   IMPORT Dialog, PboxListADT, PboxStrings;

   TYPE
      String32 = ARRAY 32 OF CHAR;

   VAR
      d*: RECORD
         insertT*: PboxListADT.T; insertPosition*: INTEGER;
         removePosition*: INTEGER;
         searchT*: PboxListADT.T; searchPosition-: String32;
         retrievePosition*: INTEGER; retrieveT-: PboxListADT.T;
         numItemsA-, numItemsB-: INTEGER;
      END;
      listA, listB: PboxListADT.List;

   PROCEDURE InsertAtA*;
   BEGIN
      PboxListADT.InsertAtN(listA, d.insertPosition, d.insertT);
      d.numItemsA := PboxListADT.Length(listA);
      Dialog.Update(d)
   END InsertAtA;

   PROCEDURE InsertAtB*;
   BEGIN
      PboxListADT.InsertAtN(listB, d.insertPosition, d.insertT);
      d.numItemsB := PboxListADT.Length(listB);
      Dialog.Update(d)
   END InsertAtB;
```

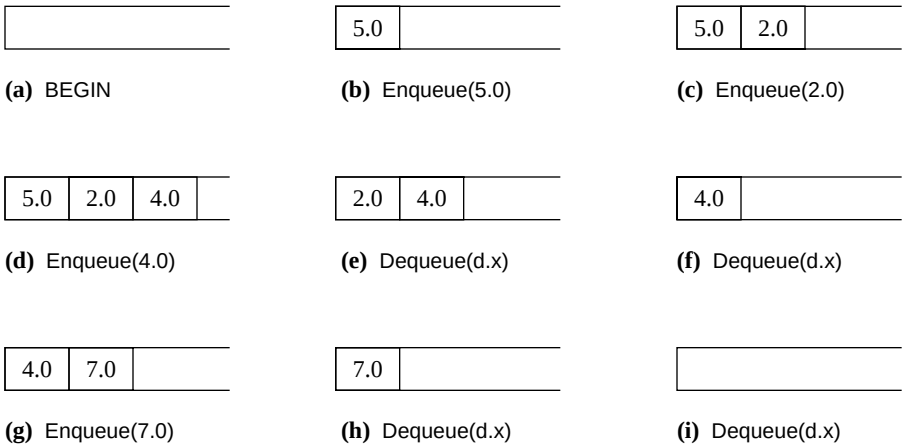
Figure 7.20
A program that uses the list abstract data type

```
PROCEDURE RemoveFromA*;
BEGIN
    PboxListADT.RemoveN(listA, d.removePosition);
    d.numItemsA := PboxListADT.Length(listA);
    Dialog.Update(d)
END RemoveFromA;

PROCEDURE RemoveFromB*;
BEGIN
    PboxListADT.RemoveN(listB, d.removePosition);
    d.numItemsB := PboxListADT.Length(listB);
    Dialog.Update(d)
END RemoveFromB;
```

```
PROCEDURE SearchForA*;
VAR
    found: BOOLEAN;
    position: INTEGER;
BEGIN
    PboxListADT.Search(listA, d.searchT, position, found);
    IF found THEN
        PboxStrings.IntToString(position, 1, d.searchPosition);
        d.searchPosition := "At position " + d.searchPosition + "."
    ELSE
        d.searchPosition := "Not in list."
    END;
    Dialog.Update(d)
END SearchForA;

PROCEDURE SearchForB*;
VAR
    found: BOOLEAN;
    position: INTEGER;
BEGIN
    PboxListADT.Search(listB, d.searchT, position, found);
    IF found THEN
        PboxStrings.IntToString(position, 1, d.searchPosition);
        d.searchPosition := "At position " + d.searchPosition + "."
    ELSE
        d.searchPosition := "Not in list."
    END;
    Dialog.Update(d)
END SearchForB;
```

```
PROCEDURE RetrieveFromA*;
BEGIN
   PboxListADT.GetElementN(listA, d.retrievePosition, d.retrieveT);
   Dialog.Update(d)
END RetrieveFromA;

PROCEDURE RetrieveFromB*;
BEGIN
   PboxListADT.GetElementN(listB, d.retrievePosition, d.retrieveT);
   Dialog.Update(d)
END RetrieveFromB;
```

```
    PROCEDURE DisplayListA*;
    BEGIN
        PboxListADT.Display(listA);
    END DisplayListA;

    PROCEDURE DisplayListB*;
    BEGIN
        PboxListADT.Display(listB);
    END DisplayListB;

    PROCEDURE ClearLists*;
    BEGIN
        PboxListADT.Clear(listA); PboxListADT.Clear(listB);
        d.insertT := ""; d.insertPosition := 0;
        d.removePosition := 0;
        d.searchT := ""; d.searchPosition := "";
        d.retrievePosition := 0; d.retrieveT := "";
        d.numItemsA := 0; d.numItemsB := 0;
        Dialog.Update(d)
    END ClearLists;

BEGIN
    ClearLists
END Pbox07D.
```



(a) BEGIN **(b)** Enqueue(5.0) **(c)** Enqueue(2.0)

(d) Enqueue(4.0) **(e)** Dequeue(d.x) **(f)** Dequeue(d.x)

(g) Enqueue(7.0) **(h)** Dequeue(d.x) **(i)** Dequeue(d.x)

Figure 7.21
A sequence of operations on a queue.

- IF in the client.
- ASSERT in the server.

The design-by-contract rule

- the precondition, P
- the statement, S
- the postcondition, Q

The Hoare triple

$$\{P\}S\{Q\}$$

PROCEDURE **RemoveN** (VAR lst: List; n: INTEGER)
Pre
0 <= n 20
Post
If n < Length(lst), the element at position n in list lst is removed.
Otherwise, the list is unchanged.

$\{0 \leq n \wedge Length(lst) = \mathbf{L}\}$

lst := ?

$\{(n < \mathbf{L} \Rightarrow \text{Element at } n \text{ is removed} \wedge Length(lst) = \mathbf{L} - 1) \wedge (n \geq \mathbf{L} \Rightarrow lst \text{ is unchanged})\}$

PROCEDURE **Length** (IN lst: List): INTEGER
Post
Returns the number of elements in list lst.

$\{true\}S\{Length(lst) = \text{The length of } lst\}$

$\{R[x := E]\}\, x := E\{R\}$

Formal definition of assignment